# *Optical Navigation System Simulation Guide V1.0*

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# Contents

# List of Figures

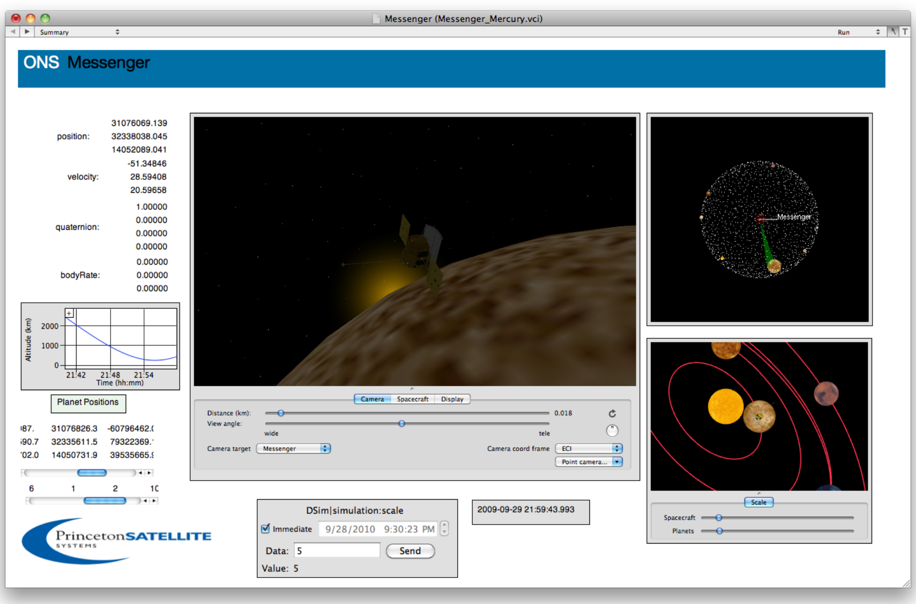
# 1 Summary

This document gives a brief introduction to running the Optical Navigation System simulation in VisualCommander. It provides two test cases that can be used as a template for developing additional simulations:

1. New Horizons Pluto flyby
   (a) A spacecraft similar to NewHorizons with ONS with a thruster control system
   (b) The spacecraft is within 1 day of Pluto encounter
   (c) The ephemeris includes the Earth, Moon, Mars, Jupiter, Saturn, Uranus, Neptune and Pluto
2. Messenger Mercury flyby Figure 1-1
   (a) A spacecraft similar to Messenger. The simulation is of the September 2009 flyby of Mercury.
   (b) The simulation starts a before the flyby
   (c) The ephemeris includes all of the planets out to Saturn

This document explains how to build the simulation and control software from subversion. That section can be skipped if you are building it from an installer. Also provided with this Users Guide is an HTML version of the API documentation for all of the software. VisualCommander has a built-in help system accessible from the help menu.

**Figure 1-1**. Mercury Messenger flyby 9/21/2009




# 2 Simulation Components

Table 2-1 on the next page lists the software components provided with this package. Running the complete simulation requires a computer running Mac OS 10.6+.

The icons for VisualCommander and DSimManager are shown in Figure 2-1 on the following page.

**Table 2-1**. Simulation components

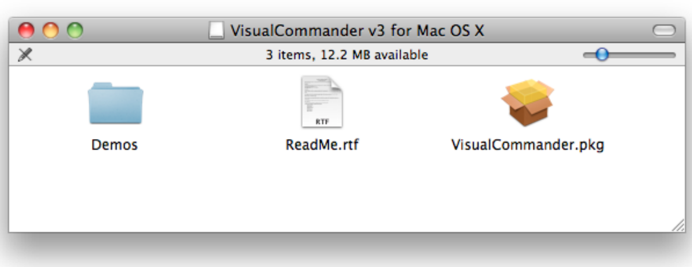
| Component | Description |
|---|---|
| VisualCommander v3.0 | The client/server application for running simulations, interacting with the simulation and visualizing the simulation. The client portion of VC 3.0 requires Mac OS 10.6+. |
| DSim | The cross-platform simulation engine |
| Spacecraft Control Library (SCControl) | A library of C/C++ classes and functions for many common spacecraft simulations |
| Spacecraft Package | A library of C++ classes for DSim spacecraft simulations |
| DSimManager v1.0 | An application for building DSim simulations |
| ControlDeck v2.0 | A C++ library for building real-time control systems |
| DCS | An Xcode project that contains all ControlDeck software that implements the system |
| APIs | Doxygen APIs are provided for ONS, the Spacecraft package and SC-Control |

**Figure 2-1**. Icons


# 3 Building from an Installer

The installer is shown in Figure 3-1. Double click on it and it will install VisualCommander. If you see VisualCommander in the toolbar it is preinstalled.

**Figure 3-1**. Installer

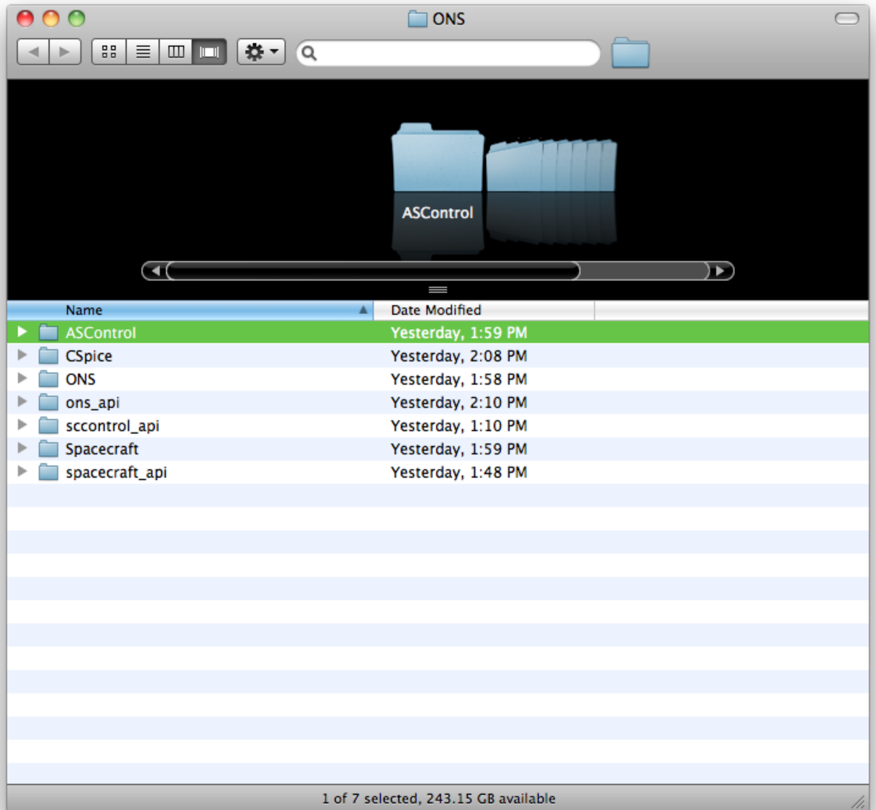



Some components need to be copied onto the computer. Those are shown in Figure 3-2 on the following page. These include the Xcode projects and the APIs.

# 4 Building from SVN

If you have access the the Princeton Satellite Systems Subversion (svn) archive you can build your simula-

**Figure 3-2**. ONS folder




tion and control deck from svn. You need to build the following:

1. Frameworks - frameworks consists of many different Xcode projects. You will not normally need to look at any of these except for ASControl which includes many C++ functions related to dynamics and control
2. Spacecraft - this contains all the DSim dynamical models
3. ONS - Optical Navigation System ControlDeck. These are all the control and navigation functions

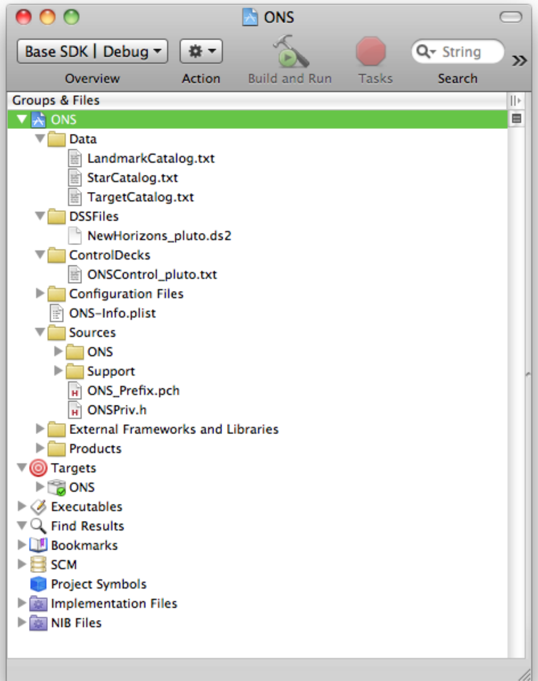
All of the packages are found on the PSS svn server. The following commands will download the required software

```
svn co https://svn.psatellite.com/Packages/ONS
svn co https://svn.psatellite.com/Packages/Spacecraft
svn co https://svn.psatellite.com/Packages/CSpice
svn co https://svn.psatellite.com/VisualCommander
svn co https://svn.psatellite.com/Frameworks
```

The ONS Xcode project is shown in Figure 4-1 on the next page. The window shows the major folders in the project. It also shows all the libraries needed to build the ONS software.

You need to first build Frameworks. Building frameworks is done using Terminal. An example session is shown in Figure 4-2 on the following page. The sessions show all of the projects that are built. The first

**Figure 4-1**. ONS project



command, “make macclean" cleans out all the compiled code. “make macrelease" creates a release version of each package.

To build an Xcode project double click on the project file, `xxx.xcodeproj`. Under Build select “Clean all Targets". Once this is done select “Build"

# 5 Architecture of the Simulation

Figure 5-1 on page 9 shows how the user interacts with VisualCommander. The user interacts with the simulation via DSimManager, a self-contained application, and with the ControlDeck, which contains all of the control and navigation software and its interface with the simulation. The sequence of steps is to

1. Build a simulation using DSimManager
2. Write a ControlDeck. The ControlDeck has a link to the simulation
3. Start VC and connect the ControlDeck to a session
4. Build your interface interactively in VisualCommander

The file types are

1. .vci file - contains the user interface
2. .txt file - contains the control deck

**Figure 4-2**. Building frameworks in Terminal


3. .ds2 file - an xml file that contains the simulation definition

# 6 APIs

Figure 6-1 on page 10 shows the index page for the SCControl API. All three APIs (Spacecraft, ONS and SCControl) are in doxygen format. They are provided in 3 folders.

# 7 Demonstration

## 7.1 Introduction

This section takes you through a complete simulation of the Optical Navigation System as the New Horizons spacecraft approaches Pluto. The model has the same thruster layout as the actual New Horizons and its

**Figure 5-1**. User interaction

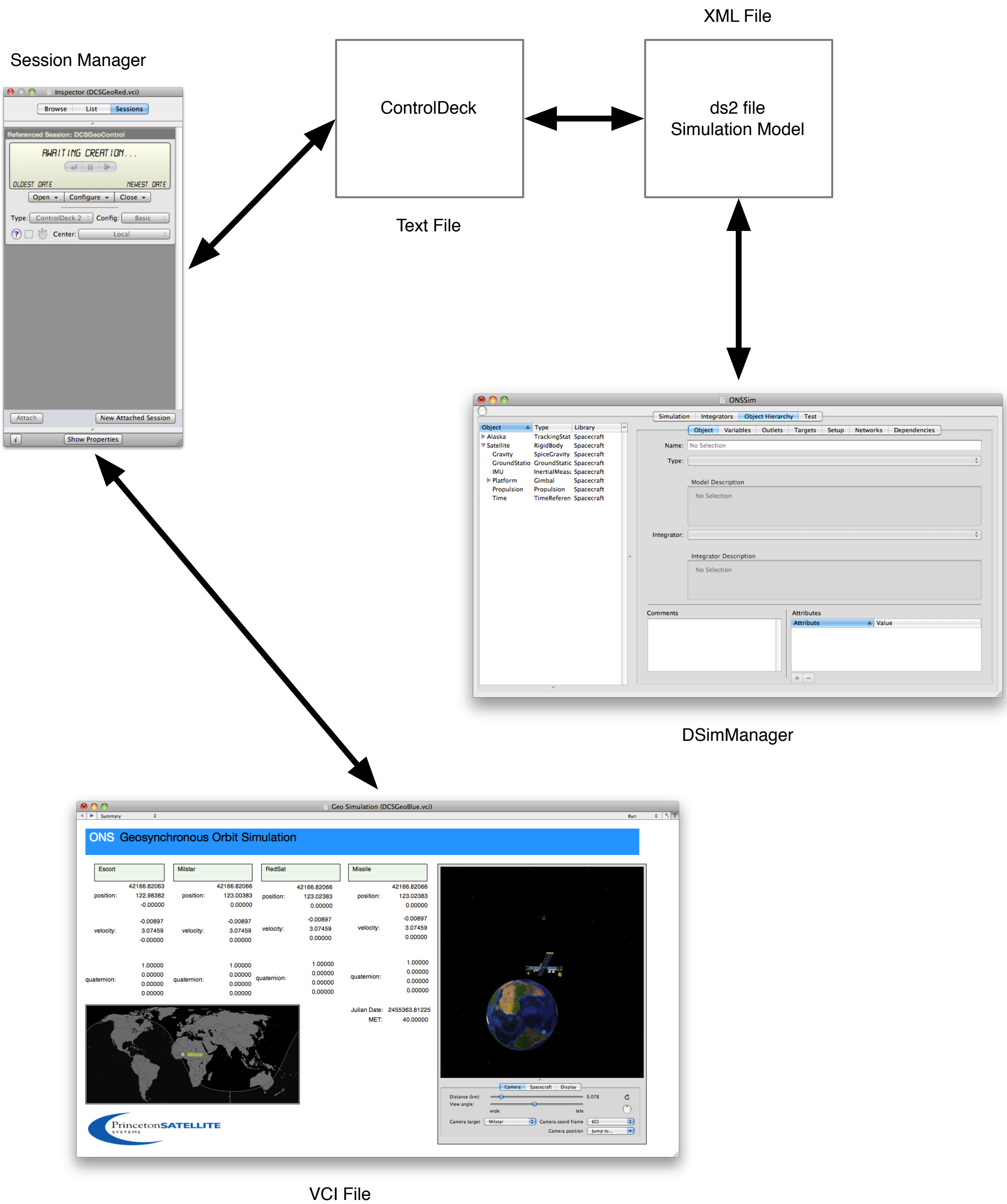
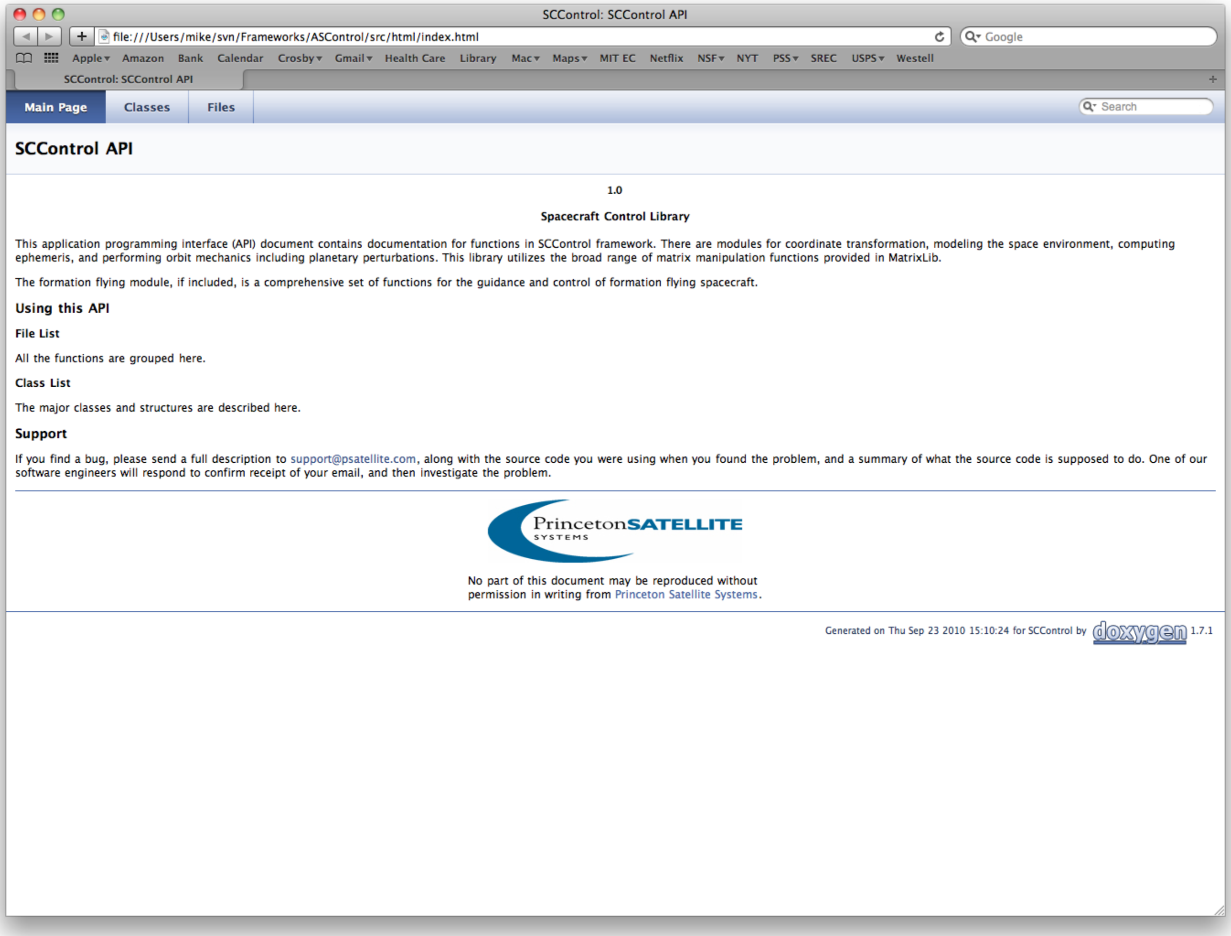


**Figure 6-1**. SCControl API index page




mass properties are similar but there is no attempt to exactly replicate the spacecraft. The situation for the Messenger simulation is similar, differing only in the details of the orbit and objects tracked.

The spacecraft does the following:

1. Acquires Pluto and Polaris using the telescopes
2. Computes the orbit
3. Computes the attitude
4. Flies past Pluto

To start the simulation find your VisualCommander application and double click on the application icon. You will need to pull down the file menu and find the file `NewHorizons_pluto.ds2`. The simulation will begin to run.

The following figures show a typical simulation. The menu in the upper left hand corner allows you to change pages so that you can view other aspects of the simulation. You can also use the arrow keys for this purpose.

**Figure 7-1**. VisualCommander simulation start


After the program starts, ONS automatically begins to align one sensor with Pluto and the other with a polar star. The attitude estimate just uses the single frame solution and is allowed to converge. The progress of the alignment can be viewed on the Summary page in the upper right frame, or on the Tracking page where the attitudes of the cameras can be followed quantitatively.

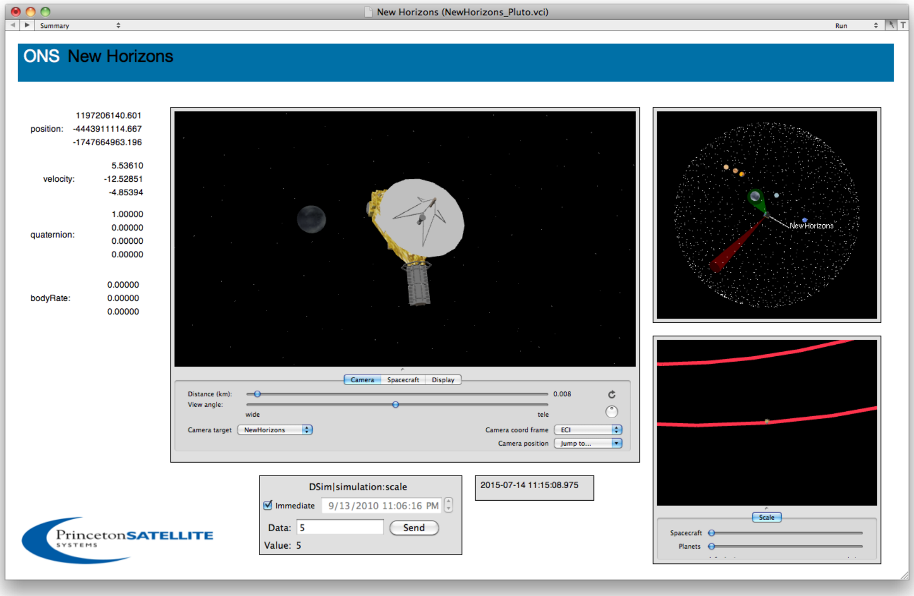
The program is initialized with the optical sensor input to the orbit determination routine turned off. The estimator runs, but has no optical input. Once the program is running, the optical sensor input can be enabled. On the Orbit Determination page enter a 1 in the od_use_camera_command entry field, and click <Send> to execute the command. With the new input from the camera, the covariance begins to decrease eventually reaching down to about 25 $km^2$. The orbit determination is accomplished using only the radius of Pluto and Pluto centroid/star centroid measurements. Figure 7-2 on the next page shows the state vector for the spacecraft (distance vector, velocity vector) on the left hand side. The large window is a 3D view of the spacecraft, planets and stars. The display in the upper right hand corner shows the sensor cones. The display below that shows the solar system and position of the planets and spacecraft. “DSim|simulation:scale" is the ratio of sim speed to real-time. Anywhere from 1 to 10 should work on most computers.

The progress of New Horizons can be monitored from this page as the spacecraft passes Pluto. In the large frame one can observe the spacecraft and Pluto in the same view, and watch as Pluto recedes after the spacecraft passes. In the upper right frame, one can observe the attitude of the sensors as they slew to keep Pluto in the field of view. It will likely be necessary to adjust the view in the frames by dragging with the mouse on the image. There are also controls below the frame to change the position and field of view of the observer.

Figure 7-3 on the following page shows the orbit determination page. When simulation starts, the position and velocity of the spacecraft are initialized, and the state of the spacecraft thereafter is determined by the physical model. The orbit determination estimator is running with access to the model, but input from the camera is turned off. On the orbit determination page one can see the results of the orbit determination routine in the variables labeled “od_state”, while in the upper right hand corner the state of the craft as determined by the simulation is shown for comparison. Also shown is the estimator’s covariance matrix.

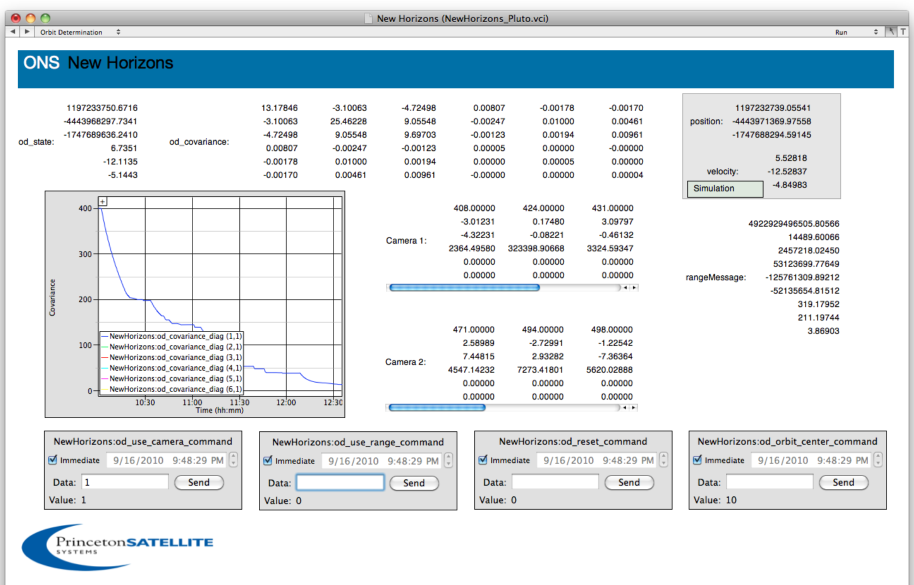
Along the bottom of the frame are displayed several commands that are used to control the orbit determination process. In particular, as mentioned above, the camera input can be turned on to add that data to

**Figure 7-2**. Simulation summary page



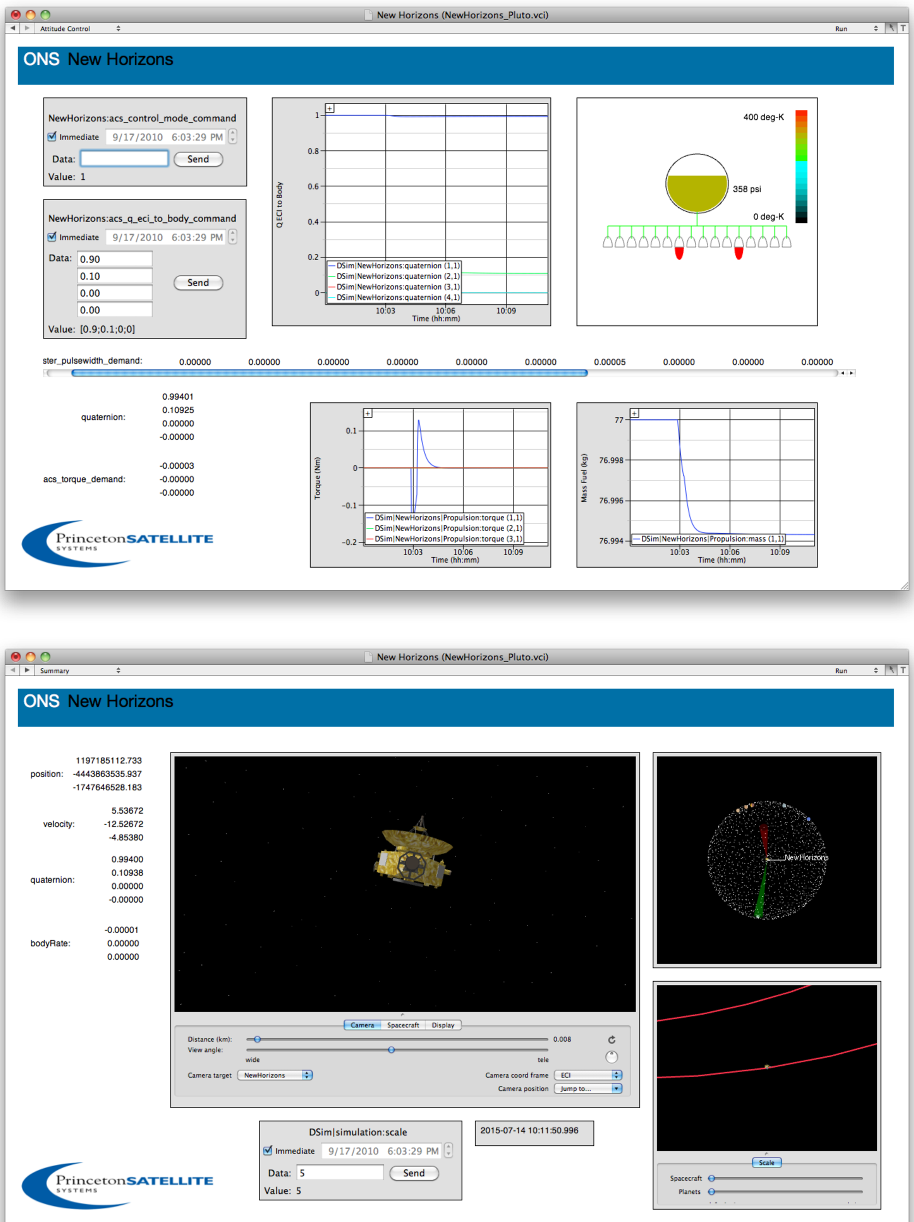

the estimator. With the camera enabled, one can observe the covariance diagonal decrease as the estimation improves with the new data. Similarly, a simulated range message from a ground station can be added by enabling "od_usd_range_command". message sent from the simulated ground station.

**Figure 7-3**. Orbit determination with Pluto as a target




You can do an attitude maneuver using the Attitude Control page. Figure 7-4 on the next page shows a roll maneuver. We waited until the targets were tracked and attitude control converged. To execute an attitude maneuver, enter the desired final attitude quaternion in the four input fields for acs_q_eci_to_body_command, and click <Send> to load the command, then enter 1 in acs_control_mode_command followed by <Send> to execute the command. (This last step may not be necessary.) Tracking maintains alignment on its targets even during the maneuver. Graphical display of thruster parameters can be viewed on the Attitude Control page, and the changing attitude of the spacecraft can be followed on the Summary page. Also on the Summary page you can watch the camera sensors realign to keep their targets in view. One advantage of ONS is that if you start with the cameras on a star field, the sensors will stay on the star field during the maneuver. Of course if the maneuver is large enough you must make sure that the tracking targets switch to new targets that can be seen by the sensor.

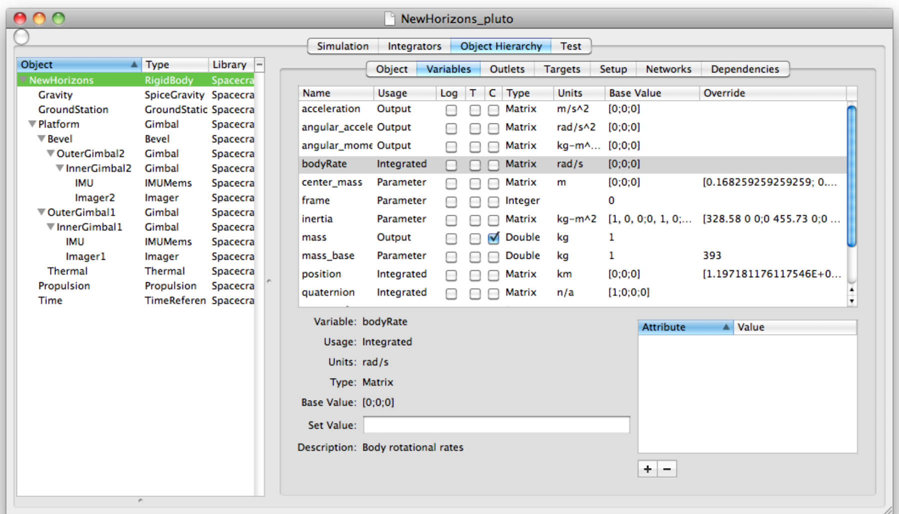
**Figure 7-4**. Attitude control maneuver



## 7.2 Simulation Setup File

DSim is the software component that actually executes the simulation. A simulation is constructed and initialized by means of a XML file, which in the case of the New Horizons simulation is in a file `NewHorizons_pluto.ds2`. There is a GUI application, DSimManager, that allows the setup file to be constructed, viewed, and modified. Modules that model simulation components can be added to the simulation by drag-and-drop, variables initialized, connected, and tagged for logging, etc. For example, a spacecraft can be added to the simulation simply by dragging it's entry in the Components window into the Object Hierarchy. The file as exposed in DSimManager is shown in Figure 7-5. The Object Hierarchy tab is open showing the spacecraft modeled in the simulation. The main spacecraft hierarchy is expanded showing all of the variables associated with the spacecraft model.

**Figure 7-5**. Simulation setup file in DSimManager




## 7.3 Control Deck

DSim has no control capability. It simply runs the simulation given the data and parameters it has received. DSim gets its data from some external source. One source is the Control Deck. The Control Deck can read data from the simulation, process data, and send commands and data to the simulation. That is, it performs analysis and control functions. The Control Deck contains a list of all of the modules (instantiated C++ classes) in the control system and creates mappings between DSim variables and variables it uses itself. The Control Deck for this demonstration is found in the file `ONSControl_pluto.txt`. The file is shown below.

**Listing 1**. Control deck *ONSControl_pluto.txt*

```
Simulation /Library/Application Support/VisualCommander/Model Libraries/NewHorizons_pluto.ds2

System NewHorizons

TimeScale 1

# Time
#----------------------------------------------------------------------------
Module NewHorizons ons_timer                    Timer                   ONS
```

```

# Time
#--------------------------------------------------------------------------
Module NewHorizons ons_time                     Time                    ONS

# Set up
#--------------------------------------------------------------------------
Module NewHorizons ons_init_nh_pluto            Setup                   ONS
Module NewHorizons ons_unused_variables         Unused_Variables        ONS
Module NewHorizons ons_communications           Communications          ONS

# Command processing module. All commands come here
#--------------------------------------------------------------------------
Module NewHorizons ons_command                  Command                 ONS

# Telemetry module. All telemetry to VC will come from this module
#--------------------------------------------------------------------------
Module NewHorizons ons_telemetry                Telemetry               ONS

# Interfaces
#--------------------------------------------------------------------------
Module NewHorizons ons_image_processing         Image_Processing        ONS
Module NewHorizons ons_gimbal_processing        Gimbal_Processing       ONS
Module NewHorizons ons_mems_imu_processing      IMU_Processing          ONS
Module NewHorizons ons_mass                     Mass_Processing         ONS

# Catalogs
#--------------------------------------------------------------------------
Module NewHorizons ons_star_catalog             Star_Catalog            ONS

# Utilities
#--------------------------------------------------------------------------
Module NewHorizons ons_ephemeris                Ephemeris               ONS
Module NewHorizons ons_thermal_control          Thermal                 ONS
Module NewHorizons ons_fault_detection          FailureDetection        ONS
Module NewHorizons ons_co_id                    Celestial_Object_ID     ONS

# Estimation
#--------------------------------------------------------------------------
Module NewHorizons ons_od                       Orbit_Determination     ONS
Module NewHorizons ons_attitude_determination   Attitude_Determination  ONS

# Control and Estimation
#--------------------------------------------------------------------------
Module NewHorizons ons_attitude_control         Attitude_Control        ONS
Module NewHorizons ons_propulsion               Propulsion              ONS
Module NewHorizons ons_targeting                Targeting               ONS
Module NewHorizons ons_gimbal_control           Gimbal_Control          ONS

#--------------------------------------------------------------------------
# Inputs
#--------------------------------------------------------------------------
Variable NewHorizons|Time:julianDate            NewHorizons jd_sim
Variable NewHorizons:quaternion                 NewHorizons quaternion_sim
Variable NewHorizons:bodyRate                   NewHorizons omega_sim
Variable NewHorizons:position                   NewHorizons position_sim
Variable NewHorizons:velocity                   NewHorizons velocity_sim
Variable NewHorizons:mass                       NewHorizons mass_sim
Variable NewHorizons:center_mass                NewHorizons center_of_mass_sim
Variable NewHorizons:inertia                    NewHorizons inertia_sim

# Ephemeris
#--------------------------------------------------------------------------
Variable NewHorizons|Gravity:observer           NewHorizons observer_sim
Variable NewHorizons|Gravity:rotationMatrixBody NewHorizons rotation_matrix_planet_sim
Variable NewHorizons|Gravity:planet_index       NewHorizons planet_index_sim
Variable NewHorizons|Gravity:positionBody       NewHorizons position_planet_sim
Variable NewHorizons|Gravity:muBody             NewHorizons planet_mu_sim
```

```
Variable NewHorizons|Gravity:radiusBody         NewHorizons planet_radius_sim

# Ground Station
#--------------------------------------------------------------------------
```

*ONSControl_pluto.txt*

The first line gives the path to the simulation file which has the suffix `.ds2`. The line with "System" defines the name of a control system, `NewHorizons`. A simulation may contain a number of control systems. `TimeScale` gives the ratio of the control period to real-time.

The lines starting with `Module` attach processing modules to ControlDeck. Each module is an instantiation of a C++ class which has the base class `cd_control_module`. For example

```
Module NewHorizons ons_ephemeris Ephemeris ONS
```

Defines a Module for the control system `NewHorizons` using the `ons_ephemeris` class. It is given the name `Ephemeris` and is from the `ONS` bundle which is built by `ONS.xcodeproj`.

The `Variable` line connects the ControlDeck to the DSim model. For example

```
Variable NewHorizons|Time:julianDate NewHorizons jd_sim
```

maps the variable `julianDate` from the `Time` module (an instance of the class `ons_timer`) which is a child of the `NewHorizons` model to the ControlDeck variable `jd_sim` in the System `NewHorizons`. Now any module within the ControlDeck can access `julianDate` via the Control Deck variable `jd_sim` with a call to the `request_data` function.

```
        jd_sim_ref = request_data( NULL, "jd_sim", sd_type_double );
```

## 7.4 VCI File

You run the simulation by selecting `NewHorizons_Pluto.vci` either from VisualCommander or double clicking on the file. The window has a number of pages that can be used to send commands to the simulation and to observe results. You can customize the pages, or add more pages, while the simulation is running by dragging tools and/or data points onto the window. See the VisualCommander online help for more information.

## 7.5 User Operations

Commands that the user can use during the demonstration are shown in Table 7-1 on the next page. The table shows the pages and the commands available on each page. You can add commands or telemetry to any page. You can see some of the commands and telemetry in Figure 7-6 on the following page.

## 7.6 Editing

Figure 7-7 on the next page shows the data flow window which can be used to connect simulation data to VisualCommander displays. You drag and drop from the menus on the left to the displays on the right.

You can customize any window even while the simulation is running. Figure 7-8 on page 18 shows an editing sequence. In this case the user is adding the raw data point "jd" to the window. You use the menu on the right to switch from run to edit mode. The simulation does not stop in edit mode but you can only send commands in run mode.

**Table 7-1**. Simulation commands

| Page | Commands |
| --- | --- |
| Summary | simulation scale - adjust the ratio of sim speed to real time. |
| Attitude Determination | Single Frame Command - select the single frame mode for attitude determination. This uses a pseudo-inverse. |
| Orbit Determination | Reset command, Use range measurement command, use optical measurement command and change orbit center command |
| Attitude Control | Send a quaternion to reorient the spacecraft. You can also change the attitude maneuver model |

**Figure 7-6**. Data window in VisualCommander

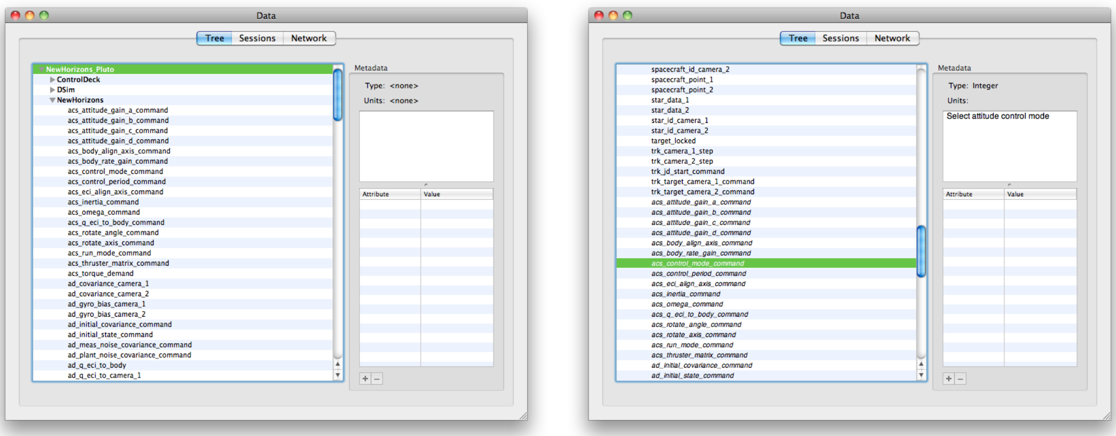



**Figure 7-7**. Data flow window in VisualCommander

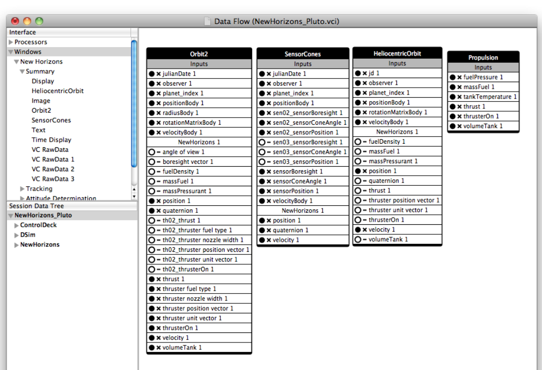
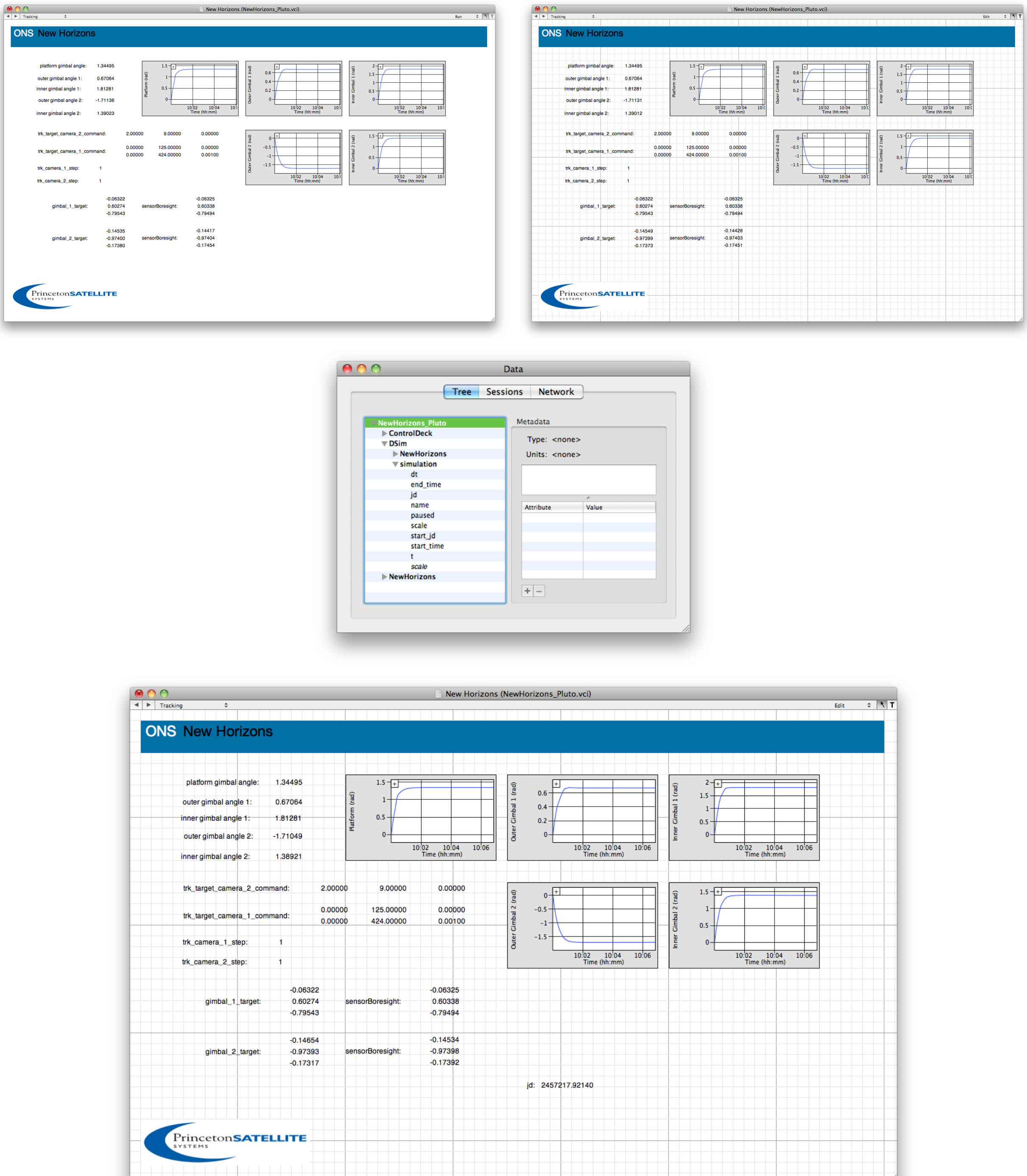


**Figure 7-8**. Editing VisualCommander



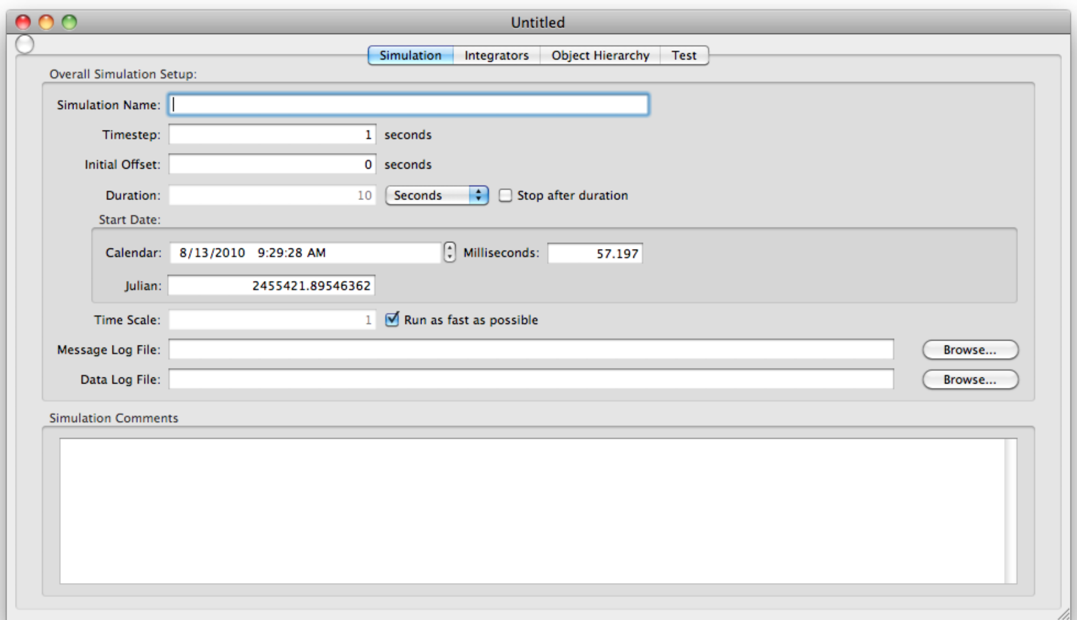
# 8 DSimManager

DSimManager is a an application that allows you to visually assemble dynamical models in the simulation. The models reside in the Spacecraft Package or any DSim class library that you may have.

When you open DSimManager two windows appear. One says “Untitled" and is the window for building simulations Figure 8-1. The second is the “Components" window which has your DSim Libraries shown in Figure 8-2 on the following page. The first step is to click on the Integrators tab and add an integrator. An integrator integrates the equations of motion, and effects the time evolution of system quantities. Hit the “+" button and give it a name. There are two built in integrators but you can add your own.

Once you’ve done that you can drag and drop models into the object hierarchy window shown in Figure 8-2 on the next page. Models are in a hierarchy. The gravity model is a child of the spacecraft model. This facilitates transferring information from one model to the other. Other mechanisms for transferring data such as targets, networks and outlets also are available, and are exposed in tabbed pages in DSimManager.

**Figure 8-1**. DSimManager




# 9 Simulation Models

## 9.1 Spacecraft Dynamics

The translational dynamics are

$$\dot{r} = v \tag{9-1}$$

**Figure 8-2**. DSimManager: Components window

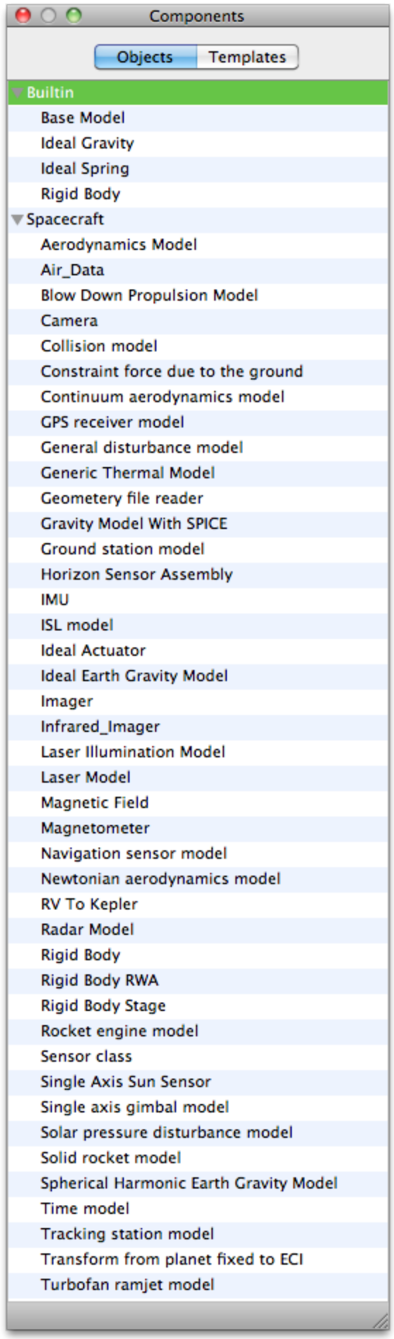


**Figure 8-3**. DSimManager: select integrator

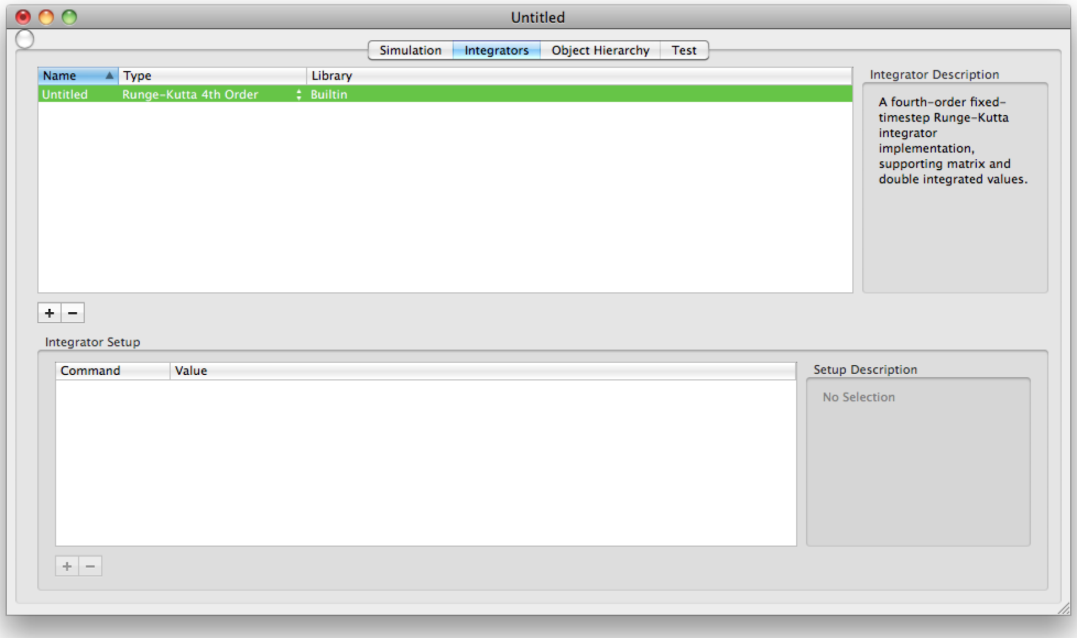
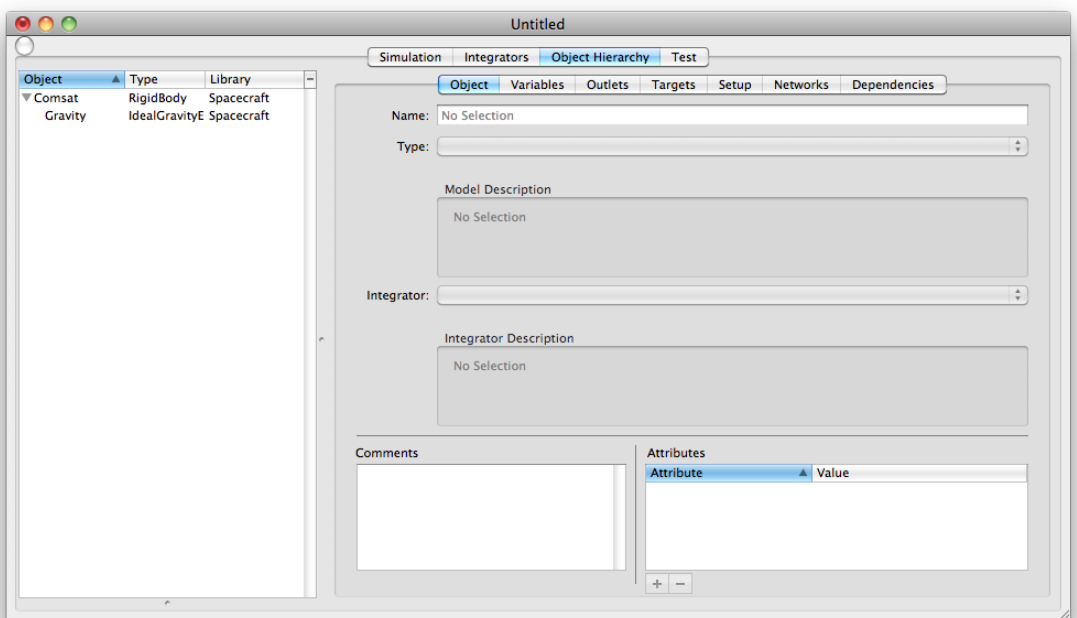



**Figure 8-4**. DSimManager: Drag and drop models



$$\dot{v} = a \quad (9\text{-}2)$$

$$\dot{m}_f = -\frac{F_t}{u_e} \quad (9\text{-}3)$$

The acceleration is

$$a = \frac{F_s + F_d + F_t}{m_d + m_f} + a_c + a_p \quad (9\text{-}4)$$

where $F_a$ is the force due to solar pressure, $F_d$ is the force due to atmospheric drag and $F_t$ is the force due to thrusters. $a_c$ is the gravitational acceleration due to the central body and $a_p$ is the acceleration due to the sum of accelerations from all other bodies. $u_e$ is the thruster exhaust velocity. $m_f$ is the fuel mass and $m_d$ is the dry mass.

The rotational dynamics and kinematics are

$$I\dot{\omega} + \omega \times I\omega = T \quad (9\text{-}5)$$

$$\dot{q} = f(q, \omega) \quad (9\text{-}6)$$

The rotational kinematics are represented by quaternions.

## 9.2 Disturbance Model

The disturbances of interest are solar pressure and atmospheric drag. The latter is only important near planets with atmospheres. The spacecraft surface is represented by a triangle mesh. Each mesh element is represented by 3 vertices a normal and the following properties

1. drag coefficient, $C_D$
2. specular reflection coefficient, $\rho_s$
3. diffuse reflection coefficient, $\rho_d$
4. absorption coefficient, $\rho_a$
5. transmissivity $\rho_t$

$C_D$ is between 0 for no drag and 4 for when the surface perfectly reflects incoming particles.

The drag is

$$F_d = -\frac{1}{2}\rho C_d A n^T v v \quad (9\text{-}7)$$

where $n$ is the outward unit normal, $v$ is the velocity in the body frame and $A$ is the area of the plate. The outward normal is

$$n = \frac{(v_1 - v_3) \times (v_2 - v_3)}{|(v_1 - v_3) \times (v_2 - v_3)|} \quad (9\text{-}8)$$

Solar pressure is the dominant disturbance on a spacecraft in geosynchronous orbit. Solar pressure is due to the force of photons on the surfaces of the spacecraft. A photon striking a surface can do one of four things: it can be absorbed, it can reflect specularly (meaning at the same angle with respect to the surface normal that it hit), it can reflect diffusely (meaning at any angle), or it can pass through the surface. Photons that are absorbed must either be transferred somewhere else (through heat conduction or as electricity in the case of a solar array) or be remitted locally. If the latter happens, the photon must be lumped in with the diffusely re-emitted photons. In terms of fractions of the incoming photons, the following is true for a surface

$$1 = \rho_a + \rho_s + \rho_d + \rho_t \quad (9\text{-}9)$$

where $\rho$ stands for the fraction of photons that are absorbed, specularly reflected, diffusely reflected or transmitted.

The solar pressure force for a surface with only one side seeing free space is

$$F = -SAs^T n(2(\rho_s s^T n + \rho_d/3)n + (\rho_a + \rho_d)s) \tag{9-10}$$

where $s$ is the Sun vector, $n$ is the unit normal to the surface, $A$ is the area of the surface and $S$ is the solar flux in N/m$^2$.

For thin membranes (such as solar sails or solar arrays) we can account for front and back simultaneously. Each side of the membrane, front and back, has its own emissivity ($\epsilon$).

In steady state the incoming absorbed solar flux must equal the outgoing re-emitted flux, which follows Boltzmann's law for thermal radiation. Assuming that the front and back temperatures of the object are the same ($T_s$) then we can write

$$\rho_a P = \sigma\left(\epsilon_f T_s^4 + \epsilon_b T_s^4\right) \tag{9-11}$$

where $P$ is the incoming solar flux, $P = \Phi\cos(\theta)$, and $\sigma$ is Boltzmann's constant. We can solve for the equilibrium temperature as

$$T_s = \left(\frac{\rho_a P}{\sigma(\epsilon_f + \epsilon_b)}\right)^{1/4} \tag{9-12}$$

The re-emitted fluxes per unit area for the front and back of the membrane are

$$\Phi_f = \frac{\epsilon_f \rho_a P}{\epsilon_f + \epsilon_b} \tag{9-13}$$

and

$$\Phi_b = \frac{\epsilon_b \rho_a P}{\epsilon_f + \epsilon_b} \tag{9-14}$$

which are independent of temperature. The remaining outgoing fluxes accounting for the specular and diffuse reflected portions of the solar flux are

$$\Phi_s = \rho_s P \tag{9-15}$$

$$\Phi_d = \rho_d P \tag{9-16}$$

From the conservation of energy we know that these four outgoing fluxes must equal the incoming flux. The total force exerted is

$$F = \cos\theta\,(f_s + f_d + f_f + f_b) \tag{9-17}$$

and the normalized force contributions due to each flux are

$$f_s = -2\cos\theta\Phi_s\hat{n} \tag{9-18}$$

$$f_d = -\Phi_d\left(\frac{2}{3}\hat{n} + \hat{s}\right)$$

$$f_f = -\frac{2}{3}\Phi_f\hat{n} + \Phi_f\hat{s} \tag{9-19}$$

$$f_b = \frac{2}{3}\Phi_b\hat{n} + \Phi_b\hat{s}$$

Combining terms we get the final form of the thermal/optical force model.

$$F = -pA\cos\theta \left( 2\left[ \rho_s \cos(\theta) + \frac{1}{3}\left( \rho_d + \rho_a \frac{\epsilon_f - \epsilon_b}{\epsilon_f + \epsilon_b} \right) \right] \hat{n} + [\rho_d + \rho_a]\,\hat{s} \right) \tag{9-20}$$

If the surface does not have a back set $\epsilon_f$ equal to $\epsilon_b$. The resulting equation is the conventional single sided surface model. The spacecraft will be subdivided into triangles. Each triangle will have different surface properties which will allow the disturbance model to handle a wide variety of spacecraft.

The pulse widths are decremented by the simulation time step after the force and torque have been applied to the spacecraft.

## 9.3 Gravity Model

### 9.3.1 Point Mass Model

The gravity model assumes a point mass model of the center and a perturbation model for all additional bodies. Any body available in the ephemeris model can be added.

The point mass model is

$$a = -\mu \frac{r}{|r|^3} \tag{9-21}$$

where $r$ is the vector from the central body to the spacecraft.

The perturbation model is

$$a = -G \sum m_j \left( \frac{d_j}{|d_j|^3} + \frac{\rho_j}{|\rho_j|^3} \right) \tag{9-22}$$

where $d$ is the vector from the spacecraft to the secondary body and $\rho$ is the vector from the central body to the secondary body, i.e. $\rho = r + d$.

The model uses the SPICE library for the planetary ephemeris. The model can be used in the ECI or ecliptic frames.

### 9.3.2 Spherical Harmonic Model

The point mass gravity field for the central body can be replaced by a spherical harmonic gravity model. There are many ways to deal with asymmetries in planets. One would be to model the planet as a set of point masses contained within the surface of the planet. The second is to expand the gravitational potential in a spherical harmonic expansion and compute the gravity forces from the gradient of the potential. Other expansions could also be used.

The spherical harmonic expansion method works as follows. Define the perturbing gravitational potential of a planet as

$$V = -\frac{\mu}{r} \sum_{n=2}^{\infty} \left[ \left( \frac{a}{r} \right)^n \sum_{m=0}^{\infty} \left( S_{n,m} \sin m\lambda + C_{n,m} \cos m\lambda \right) P_{n,m}(\sin\phi) \right] \tag{9-23}$$

defined in the planet fixed frame. $a$ is the radius of the planet.

If we define $i, j, k$ as unit vectors along the planetary $x$, $y$ and $z$ axes, the gravitational acceleration can be found as follows

$$\frac{\partial V}{\partial r} = -\sum_{n=2}^{\infty}\sum_{m=0}^{n}\frac{\partial V_{n,m}}{\partial r} \tag{9-24}$$

and

$$\begin{aligned}
\frac{\partial V_{n,m}}{\partial r} &= \frac{\mu}{r^2}\left(\frac{a}{r}\right)^n\left[-r(vH_{n,m}+B_{n,m}) + iD_{n,m} - jE_{n,m} + kH_{n,m}\right] && \text{(9-25)}\\
B_{n,m} &= (C_{n,m}\hat{C}_m + S_{n,m}\hat{S}_m)(n+m+1)P_n^m && \text{(9-26)}\\
E_{n,m} &= -m(C_{n,m}\hat{S}_{m-1} - S_{n,m}\hat{C}_{m-1})\\
D_{n,m} &= m(C_{n,m}\hat{C}_{m-1} + S_{n,m}\hat{S}_{m-1})\\
H_{n,m} &= (C_{n,m}\hat{C}_m + S_{n,m}\hat{S}_m)P_n^{m+1}\\
\hat{C}_m &= \hat{C}_1\hat{C}_{m-1} - \hat{S}_1\hat{S}_{m-1}\\
\hat{S}_m &= \hat{S}_1\hat{C}_{m-1} + \hat{C}_1\hat{S}_{m-1}
\end{aligned}$$

The starting conditions are

$$\begin{array}{ll} \hat{C}_0 = 1 & \hat{S}_0 = 0 \\ \hat{C}_1 = \frac{x}{r} & \hat{S}_1 = \frac{y}{r} \end{array} \tag{9-27}$$

The derivatives of the Legendre Polynomials are

$$\begin{aligned}
P_n^0 &= \frac{1}{n}\left[(2n-1)vP_{n-1}^0 - (n-1)P_{n-2}^0\right] && \text{(9-28)}\\
P_n^m &= P_{n-2}^m + (2n-1)P_{n-1}^{m-1} && \text{(9-29)}\\
P_0^0 &= 1\\
P_1^0 &= \frac{z}{r} \equiv v\\
P_0^1 &= 0\\
P_1^1 &= 1
\end{aligned}$$

This kind of harmonic expansion is the standard method for modeling a planets gravitational field. Harmonic models exist for the Earth, Moon, Mars and other planets. A harmonic model is convenient because it gives an idea of how much influence higher order terms have on the overall force.

## 9.4 Camera Model

The camera model is a pinhole camera has a single ray per point on the target and that ray maps to a point on the focal plane. A point $P(X, Y, Z)$ is mapped to the imaging plane by the relationships

$$u = \frac{fX}{Z} \tag{9-30}$$

$$v = \frac{fY}{Z} \tag{9-31}$$

where $u$ and $v$ are coordinates in the focal plane, $f$ is the focal length and $Z$ is the distance from the pinhole to the point along the axis normal to the focal plane. This assumes that the $Z$-axis of the coordinate frame $X, Y, Z$ is aligned with the boresight of the camera.

Two models are provided. The first is the `imager.cc` model which computes the centroids of

1. Stars
2. Planets
3. Spacecraft

In addition it computes the location of landmarks on planets and points on spacecraft. These are output as a matrix for processing by the estimation software. This bypasses processing of the images to facilitate testing of the estimators. The centroids of the planets are their centers as used by the orbit dynamics model.

The second model is the `camera.cc`. This creates a synthetic scene using the OpenGL pinhole camera model. The frame can be processed by image processing to compute the pixel plane locations of the centroids.

## 9.5 IMU Model

The gyro model is

$$\omega_m = \omega + c + b + \nu_\omega \tag{9-32}$$
$$\dot{b} = -\frac{1}{\tau}b + \nu_b \tag{9-33}$$

where $c$ is a constant error, $b$ is a bias and $\nu_\omega$ is output noise. $\nu_b$ is the random walk noise. $\tau$ is the time constant for the bias. This would cause the bias to decay with time if not driven by white noise. The accelerometer model is

$$\ddot{x}_m = \ddot{x} + c + b + \nu_{\ddot{x}} \tag{9-34}$$
$$\dot{b} = -\frac{1}{\tau}b + \nu_b \tag{9-35}$$

where $c$ is a constant error, $b$ is a bias and $\nu_\omega$ is output noise. $\nu_b$ is the random walk noise.

## 9.6 Ranging Model

The ranging model returns range and range-rate between a planet fixed point and the spacecraft.

$$\rho = \sqrt{r_x^2 + r_y^2 + r_z^2} \tag{9-36}$$
$$\dot{\rho} = \frac{v^T r}{\rho} \tag{9-37}$$

## 9.7 State Sensor Model

The state sensor `state_sensor.cc` is useful for testing purposes. It returns the vehicle rigid-body state in a single vector.

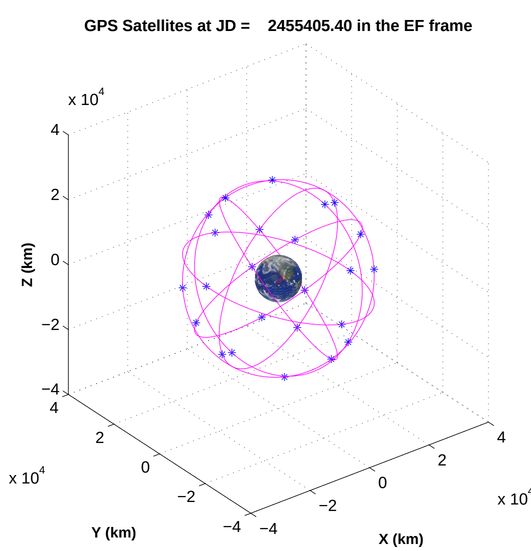
$$x = \begin{bmatrix} r_x \\ r_y \\ r_z \\ v_x \\ v_y \\ v_z \\ q_s \\ q_x \\ q_y \\ q_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} \quad (9\text{-}38)$$

The zero quaternion has $q_s = 1$.

## 9.8 GPS Model

The GPS model returns range and range rate between all visible GPS satellites. It also includes their Cartesian state vector. The GPS satellite positions propagated assuming that their orbits are circular. Line of sight is compute for each satellite with a selectable angle for blocking the signal. This model is not a highly accurate model of the GPS constellation but is sufficient for software testing. The GPS constellation in the model is shown in Figure 9-1. The model also accounts for the direction of the GPS constellation. Thus when a satellite is above the constellation altitude it will only see satellites that are on the other side of the earth but not blocked by the earth.

**Figure 9-1**. GPS model constellation

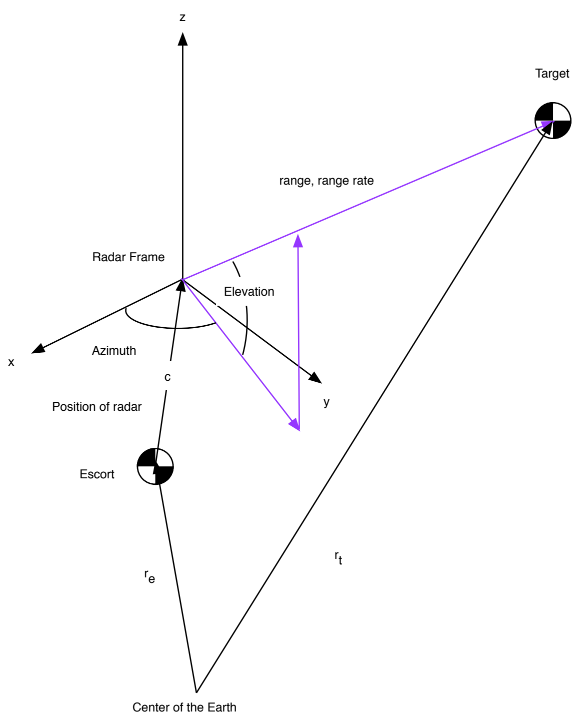


### 9.9 Intersatellite Link Model

The Intersatellite Link (ISL) model returns range and range rate between the target satellite and the core satellite. It also includes the target satellite Cartesian state vector.

### 9.10 Radar Model

**Figure 9-2**. Radar frame of reference




If the transformation matrix from ECI to the escort frame is $m_{eb}$ and the transformation matrix from the body to the radar frame is $m_{eb}$ then the relative position vector is

$$r = m_{br}\left(m_{eb}(r_t - r_e) - c\right) \tag{9-39}$$

where $c$ is the vector from the spacecraft center-of-mass to the origin of the radar frame. The gravitational equations are referenced to the center-of-mass of each spacecraft. The relative rate vector is

$$\dot{r} = m_{br}\left(\dot{m}_{eb}(r_t - r_e) + m_{eb}(\dot{r}_t - \dot{r}_e)\right) \tag{9-40}$$

if we assume that $c$ and $m_{br}$ are fixed.

Range rate is determined from the Doppler shift of the signal so it is a separate measurement from range. Azimuth and elevation are from the measured angles of the radar beam. Their rates are not directly measured. If $\alpha$ is azimuth and $\beta$ is elevation then

$$\alpha = \tan^{-1}\frac{r_y}{r_x} \tag{9-41}$$

$$\beta = \sin^{-1}\frac{r_z}{\rho} \tag{9-42}$$

where $\rho = |r|$.

## 9.11 Propulsion Model

The propulsion model allows an array of thrusters to be modeled with independent thrust parameters. A single tank blowdown system is assumed. The fuel pressure for the unregulated blowdown system is

$$P = \frac{m_{\text{He}} R_{\text{He}} T}{V - m_f/\rho_f} \tag{9-43}$$

where $m_{\text{He}}$ and $R_{\text{He}}$ are the mass and gas constant of the Helium pressurant, $T$ is the fuel temperature, $V$ is the tank volume, and $m_f$ and $\rho_f$ are the mass and density of the hydrazine fuel.

The thrust commands are pulse widths to each thruster. Any pulse width commands less than the thruster minimum impulse bits are ignored. The remaining commands are scaled if the pulse width is less than the simulation time step, so that the correct total force is applied to the spacecraft.

$$k = \left[ \begin{array}{ll} \Delta t_c/\Delta t_{sim}, & \Delta t_c < \Delta t_{sim} \\ 1.0, & \text{otherwise} \end{array} \right. \tag{9-44}$$

The thrust of each thruster is computed from a coefficient $a_0$ and the ratio of the pressure to the thruster nominal pressure.

$$\mathcal{T} = a_0 k \frac{P}{P_0} \tag{9-45}$$

The mass flow is computed from the thrust and exhaust velocity of each thruster.

$$\dot{m} = -\sum \frac{\mathcal{T}_i}{u_{e_i}} \tag{9-46}$$

The total force is a sum of the thrust times each thruster's unit vector. The torque is the cross product of the force with the vector to the spacecraft center of mass.

$$F = \sum \mathcal{T}_i \hat{u}_i \tag{9-47}$$

$$T = \sum (\vec{r} - r_{CM}) \times F \tag{9-48}$$

## 9.12 Gimbal Model

The gimbal model is a first order model in which the input command is commanded rate

$$\dot{\theta} = \omega_c \tag{9-49}$$

The gimbals may be stacked on top of other gimbals. The model does not include dynamical effects of gimbal motion. This is a reasonable simplification because the camera assembly has much lower inertia than the core spacecraft.